WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 5th Semester Examination, 2022-23

MTMACOR11T-MATHEMATICS (CC11)

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* questions from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Form the partial differential equation by eliminating arbitrary functions from the following relation:

$$z = \phi(x+iy) + \psi(x-iy)$$

(b) Solve the following partial differential equation:

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = z$$

(c) Classify the partial differential equation (elliptic, parabolic, or hyperbolic)

$$\frac{\partial^2 u}{\partial x^2} - 5\frac{\partial^2 u}{\partial x \partial y} + 6\frac{\partial^2 u}{\partial y^2} = 0$$

(d) Find the order and degree of the partial differential equations:

(i) $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial x \partial y} + \dfrac{\partial u}{\partial x} - \dfrac{\partial u}{\partial y} = 0$

(ii) $\sqrt{1 + \dfrac{\partial^2 z}{\partial y^2}} = a\left(\dfrac{\partial z}{\partial x}\right)$

(e) Form the PDE by eliminating a, b, c from $z = a(x+y) + b(x-y) + abt + c$

(f) State whether the following statement is true or false with reason:
The PDE $x(y+z)p - y(z+x)q + z(x+y) = 0$ is quasi-linear.

(g) Prove that $pv = h$ in a central orbit, where the symbols have their usual significance.

(h) A point moves along the arc of a cycloid in such a manner that the tangent at it rotates with constant angular velocity. Show that the acceleration of the moving point is constant in magnitude.

(i) A comet describes a parabola about the Sun. Prove that the sum of the squares of its velocities at the extremities of a focal chord is constant.

2. (a) Find the integral surface given by the equation
$x(y^2+z)p - y(x^2+z)q = (x^2-y^2)z$ which contains the straight line $x+y=0,\ z=1$. 5

(b) Find a complete integral of $z = px + qy + p^2 + q^2$. 3

3. Solve by the method of separation of variables: 8

$$4\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} - 3z = 0,$$ given that $z = 3e^{-y} - 3e^{-5y}$ when $x = 0$.

4. (a) Reduce the partial differential equation $yu_x + u_y = x$ to canonical form and obtain general solution. 4

   (b) Obtain the solution of the quasi linear p.d.e. $(y-u)u_x + (u-x)u_y = x-y$ with conditions $u = 0$ on $xy = 1$ using characteristic equation. 4

5. Solve the one-dimensional wave equation: 8

$$\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = 0,\ t > 0$$

subject to the boundary conditions $u(0, t) = 0,\ u(L, t) = 0,\ t > 0$ and the initial conditions $u(x, 0) = f(x),\ u_t(x, 0) = g(x)$.

6. (a) Find the differential equation of all surfaces of revolution having z-axis as the axis of revolution. 4

   (b) Find the characteristics of the equation 4

$$y^2 \frac{\partial^2 z}{\partial x^2} - x^2 \frac{\partial^2 z}{\partial y^2} = 0$$

7. Solve the Laplace's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, subject to the condition $u(0, y) = u(l, y) = u(x, 0) = 0$ and $u(x, a) = \sin\frac{n\pi}{l}x$ in $0 \le x \le l,\ 0 \le y \le a$. 8

8. A particle of mass m moves under a central attractive force $m\mu(5r^{-3} + 8c^2r^{-5})$ and it is projected from an apse at a distance c with a velocity $\frac{3\sqrt{\mu}}{c}$. Prove that the orbit is $r = c\cos\frac{2}{3}\theta$. Show further that it will arrive at the origin after a time $\frac{\pi c^2}{8\sqrt{\mu}}$. 8

9. A particle is projected with a velocity v from the Cusp of a smooth cycloid whose axis is vertical and vertex downwards, down the arc. Show that the time of reaching the vertex is 8

$$2\sqrt{\frac{a}{g}}\tan^{-1}\left(\frac{1}{v}\sqrt{4ag}\right)$$

10. The volume of a spherical raindrop falling freely increases at each instant by an amount equal to μ times its surface area at that instant. If the initial radius of the drop be 'a', then show that its radius is doubled when it has fallen through a distance $\frac{9a^2g}{32\mu^2}$. 8

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